# Irradiation modelling made simple: the cumulative sky approach and its applications

**Darren Robinson and Andrew Stone**

BDSP Partnership Ltd, Summit House, 27 Sale Place, London W2 1YR, UK.

ABSTRACT: The ability to map annual irradiation onto the urban fabric can be a powerful tool for identifying areas in which solar energy may potentially be utilised, as well as where some means of protection may be required to limit solar exposure. However, producing such output from hourly simulations, or some statistical subset of sun hours, is computationally expensive. To overcome this obstacle, this paper describes a new technique which produces annual irradiation images from a single simulation using the popular ray-tracing tool RADIANCE. This simply involves the use of a pre-processor which, given a climate file, generates a cumulative sky radiance distribution that the ray tracing program can reference at run time. The cumulative sky may be described either in terms of a global radiance distribution for a discretised sky vault, or a diffuse discretised radiance distribution with either hourly or a statistical subset of suns. Results from these different approaches are compared in terms of accuracy and speed. Some interesting alternative applications of the technique are also presented, including analysis of indoor solar penetration and temporal mean indoor / outdoor daylighting.

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## INTRODUCTION

Whether at the scale of the individual building or that of the urban neighbourhood, images of annual surface irradiation can be a powerful design tool.

At the scale of the building the quantification of solar irradiation over a given duration may help with identifying locations with good solar energy availability (and associated energy conversion potential) as well as of high solar exposure (where some form of shading may be required). Subsequent analysis of solar penetration may help with optimising façade shading in terms of reducing summertime overheating risk whilst utilising passive solar gains during the heating season.

At the urban scale municipalities may make a-priori judgements regarding potential locations for supporting the installation of solar energy conversion plant, whilst urban designers may evaluate the solar availability implications of alternative site layouts – in terms both of building energy savings and of outdoor comfort. Loss of energy to neighbouring buildings may also be quantified.

Two alternative techniques for modelling annual irradiation using the popular ray tracing program RADIANCE were launched at PLEA2000. Compagnon et al [1] describe the aggregation of results from hourly simulations of solar irradiance throughout a pre-defined grid of points (or virtual pyronometers) to produce synthesised output such as histograms of irradiation as a function of built area. Whilst useful for comparing alternative schemes, this approach lacks direct diagnosis of the location of surfaces with high/low solar exposure. Mardaljevic et al [2] on the other hand produce images of irradiation by processing individual direct and diffuse solar irradiance images for a statistical subset of sunup hours. Although this approach is relatively computationally expensive, despite the data reduction techniques used, it does provide output that is rich in diagnostic information.

This paper describes an alternative approach, providing the richness of output of [2] but at a fractional of the computational cost and using an easy to use pre-processor. The approach may also be applied to improve the computational efficiency of [1].

## 2. MODELLING METHODOLOGY

The ray tracing program RADIANCE includes a series of functions (using the module *GenSky*) to generate continuous sky luminance distributions, for example using the CIE definitions for clear, intermediate and overcast skies. When a ray intersects the sky vault the coordinates are parsed to the sky function which returns the associated luminance / radiance. An additional module *GenDaylit* produces a sky luminance distribution function using the Perez all weather model [3]. In general these functions are based on a mathematical representation of the underlying physics describing the *instantaneous* brightness distributions. In the absence of a physically-based representation it is not straightforward to produce a *continuous cumulative* sky luminance / radiance distribution. It is, however, relatively straightforward to produce a cumulative distribution for some *discretised* representation of the sky vault – so that one simply aggregates values at discrete points within the vault. We have adopted this approach in the development of a new module which we call *GenCumulativeSky*.

This simply involves discretising the sky vault using the scheme due to Tregenza in which each of 145 patches subtends a similar solid angle. The Perez all weather luminance distribution model [3] is then used to predict the luminance / radiance at the centroid of these patches and the results are aggregated for the period of interest (See Appendix).

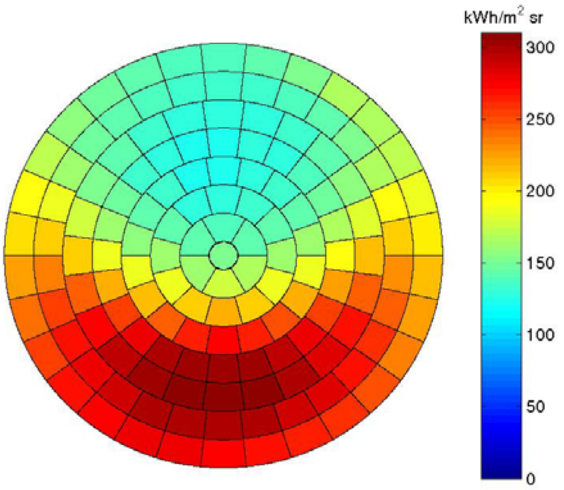


Figure 1 Cumulative diffuse sky radiance distribution for Oslo (based on 10yr mean solar data).

All that remains then is to consider the contribution due to the sun. When using instantaneous sky luminance distributions (*GenSky* and *GenDaylit*) the solar influence is provided in the form of a solar disc (of half angle 0.2665$^o$). When simulating irradiation on the other hand, we have several options – in descending order of accuracy and computational expense[1]:

(i) to represent a solar source for each point in time at which we have radiation data (Fig 2 left),
(ii) to represent solar sources within discrete regions of the sky and aggregate the solar radiance accordingly (Fig 2 right),
(iii) to increase the radiance of our sky patches in proportion to the cumulative solar energy within the patch (giving a global radiance distribution).

Again the solar contribution would be best represented by a continuous contribution (as a band) – because the sun traverses the sky smoothly rather than in 'jumps' – but as noted above this is rather more complex than simple descretisation.

[1] Although the first is not considered in this paper, as the large increase in computational cost delivers only a small increase in accuracy.

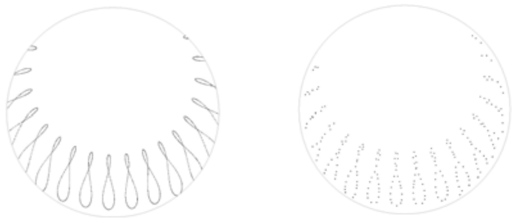

Figure 2 Solar sources for the case of hourly (left) and binned (right) sun positions.

In common with *GenDaylit*, *GenCumulativeSky* is a Program written in C. Given a set of commands specifying the climate file and location and describing the type of calculation to be performed, this produces a '.rad' file describing the size, position and radiance of the sun(s) and a reference to a separate '.cal' file which describes the sky radiance distribution in the form of a 145 element look-up table.

## 3. COMPARISONS

In order to determine the relative accuracy of the methods used a "truth result set" has been prepared. For this we have performed 1326 hourly simulations for a simple canyon and combined the resultant images (3 parallel views x 4342 daylight hours). The resultant falsecolour plot is shown in Figure 3.

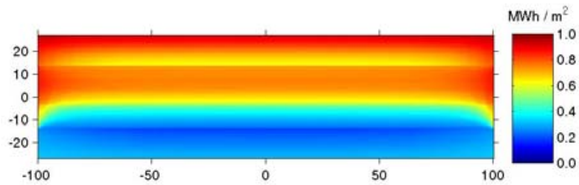


Figure 3 Annual irradiation throughout a 27m wide x 200m long canyon of h/w=1/2, located in Oslo (N.B. x, y axes relate to distance from centre of canyon).

Figure 3 is a view of a canyon that has been unfolded in the manner indicated by Figure 4 – so that the upper section relates to a south facing façade, the central section to the canyon floor and the lower section is a north facing wall.

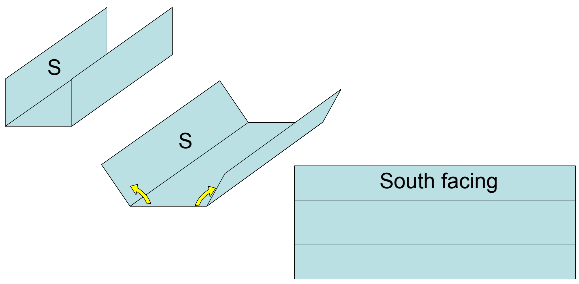


Figure 4 Unfolding of the Oslo canyon.

In producing Figure 5 below the sky is divided into 5$^o$ azimuth / altitude bins. If the sun position for a given hour has coincided with one of these bins then a solar source is defined with this bin – its radiance is cumulated for each such occurrence and its position is radiance-weighted.

It is clear from the percentage difference image that the errors associated with binning the sun position are generally very small (the maximum angular displacement from a real sun position is < $5\sqrt{2} \approx 7.08^{o}$, but it will typically be considerably lower) so that the RMS error is just 1.2%.

However, this angular displacement error is sufficient to cause the regions of under prediction towards the ends of the base of the north facing walls – as the angular position errors cause a relatively large error in the cosine of the angle of incidence within a region of generally low radiance sky.

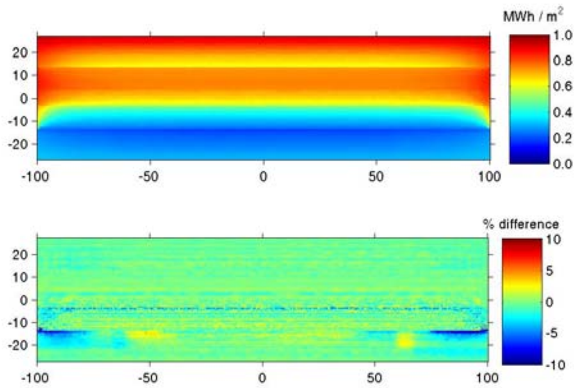


Figure 5 Annual irradiation for a diffuse sky with binned suns (upper) and associated difference plot (lower)

In uniformly distributing the cumulative solar contribution throughout entire sky patches, there will undoubtedly be occasions in which views to partially occluded sky patches ‘see’ a region of sky that has either too much or too little energy (the maximum angular displacement from a real sun position of a region of sky that can be seen is $(15^2 \times 12^2)^{1/2} \approx 19.1^{o}$). The percentage difference image in Figure 6 shows such artifacts. The strip slightly South of the centerline of the canyon floor corresponds to a region that sees a row of sky patches in which the cumulative radiance is artificially high. The corresponding RMS error for the image is 2.2%.

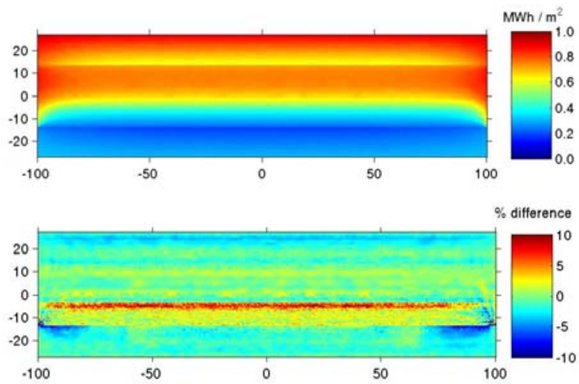


Figure 6 Annual irradiation for a global sky (upper) and associated difference plot (lower)

Finally, table 1 compares RMS error with relative run time (based on identical simulation settings) in producing the results presented in Figures 4 to 6. Note that comparisons without solar sources (diffuse-only) show that the RMS error due to discretising the sky vault is 1.02%, so that the remaining error in table 1 is due to the representation of the solar contribution.

The general benefit in utilizing the cumulative sky approach in-lieu of hourly simulations – or indeed some statistical sub-set – is clearly apparent.

Table 1 Results summary

| Method | RMS error, % | Relative run time |
|---|---|---|
| Hourly simulations | 0 | 1 |
| Diffuse discretised sky, binned suns | 1.2 | 1/180 |
| Global discretised sky | 2.2* | 1/1130 |

*By quadrupling the number of patches and increasing the simulation settings so that the patches are adequately sampled the relative run time increases to ~1/235 for a 1.55% RMS error; the over prediction strand observed in Figure 6 is also removed.

Given the relatively small simplification errors involved with using the cumulative global radiance distribution approach, it is recommended that the run time savings are sufficient to warrant its general application for irradiation modeling studies. However, there may be cases for which the importance of absolute accuracy outweighs computational efficiency – for example when making investment decisions regarding the placement of PV. In these situations it is suggested that the cumulative radiance distribution with binned solar sources should be used.

## 4. APPLICATIONS

As noted above, whether seasonal or annual, falescolour images of annual irradiation are useful in identifying areas of high solar exposure which may require some form of shading. In figure 7 for example, we have used irradiation images to help develop orientation-sensitive façade designs as part of a competition for the design of a new multi-use development within Hastings UK.

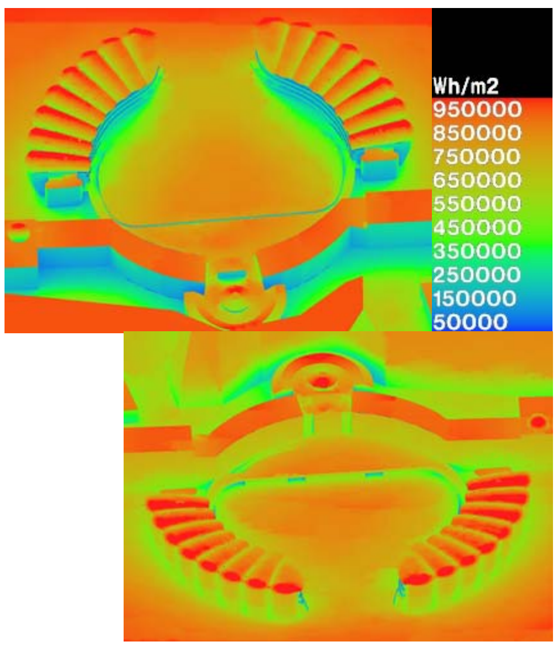


Figure 7 Annual irradiation falsecolour plot to identify surfaces with high solar exposure

As noted earlier, such images may also be used to identify locations which may be suitable for installing some form of solar collector (PV or a solar air/water heater). It has been suggested [1] that surfaces with an annual irradiation above 0.8MWh/$m^2$ may be considered viable for the installation of PV. To help with identifying such surface, an image processing toolkit has been used to re-colour (to black) pixels that are below this value (Figure 8).

Figure 8 Annual irradiation falsecolour plot, thresholded to identify surfaces with good PV potential.

Having identified locations where some form of shading device may be required subsequent analysis of solar penetration may help with optimizing the effectiveness of alternative proposals. Figure 9 (left) for example shows annual solar irradiation within a school located in London (the classrooms to the left are served by an open corridor within an atrium – the base of which is also used as a class space). As a refinement to such analyses one might instead simulate using summer and winter cumulative sky descriptions to contrast summertime solar protection with winter time solar utilisation.

An alternative way of approaching this issue is to understand the frequency of occurrence of a direct insolation event; whether from the perspective of solar exposure or glare. For example, figure 9 (right) – which depicts the number of hours during the year for which a horizontal surface receives solar radiation – has helped to secure the investment in shading at the top of the school atrium, to limit pupils' solar exposure. This has been produced by simulating cumulative solar sources (and no sky) which are scaled to give unit incident irradiance ($R_S = (\Phi_S \sin \gamma_S)^{-1}$).

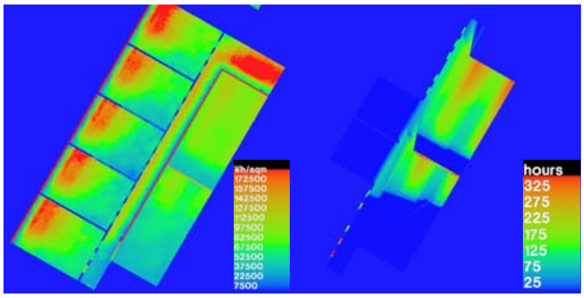


Figure 9 Annual solar penetration into school classrooms (left) and hours of solar exposure at base of school atrium (right)

Finally, by dividing the sky patch values by the number of daylight hours, a *mean* rather than *cumulative* sky may be described. This can be used, by calculating a sky *luminance* rather than *radiance* distribution, to produce temporally (rather spatially) average daylight predictions. Figure 10 for example illustrates temporal average daylight levels on a pixel-by-pixel basis throughout a proposed masterplan. This is useful in identifying external spaces which on average, under real sky conditions, may be either well or poorly daylit.

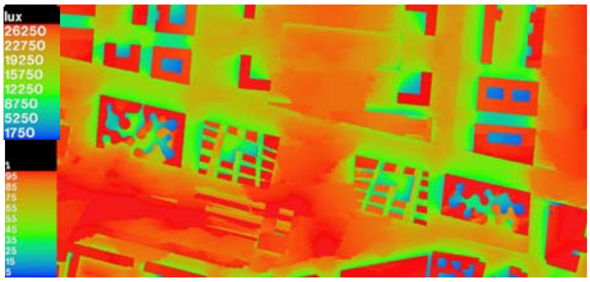


Figure 10 Temporal mean daylight levels throughout a competition stage masterplan.

## CONCLUSIONS

A new technique for simulating solar irradiation and related quantities has been developed. Called *GenCumulativeSky* this is simply a module which, given a (solar) climate file and site coordinates, pre-processes a description of a cumulative diffuse radiance / luminance distribution throughout a discretised sky vault. Two means for handling the direct solar contribution have been tested (explicit representation of actual / binned sources or assigning the contribution to sky patches to give a global radiance distribution). Of these it is felt that the latter represents the best balance between computational efficiency and accuracy – it is a factor of six faster than the binned suns approach, but with a less than two-fold increase in RMS error (to a modest *c*.2%).

Examples of the application of *GenCumulativeSky* have demonstrated its use in rapidly and simply:

- Quantifying annual solar irradiation in the urban context and identifying, in a precise way, areas with good PV potential.
- Quantifying the relative effectiveness of solar shading options in reducing solar penetration.
- Identifying the frequency of occurrence of insolation events – either internally or externally.
- Evaluating temporal mean daylight levels within the urban context (though this is equally applicable for internal analyses).

Future refinements to the technique might include the use of a finer sky discretisation and some form of bilinear interpolation to produce a sky with a relatively continuous distribution from a discretised representation.

## REFERENCES


[1] Compagnon, R. Raydan, D. (2000), *Irradiance and illuminance distributions in urban areas*, Proc. PLEA 2000, Cambridge UK, July 2000, p436-441.

[2] Mardaljevic, J. Rylatt, M. (2000), *An image-based analysis of solar radiation for urban settings*, Proc. PLEA 2000, Cambridge UK, July 2000, p442-447.

[3] Perez, R., Seals, R., Michalsky, J., (1993), *All-weather model for sky luminance distribution – preliminary configuration and validation*, Solar Energy Vol. 50 No. 3, p235-243.

[4] Perez, R., Ineichen, P., Seals, R., Michalsky, J., Stewart, R., (1990), *Modelling Daylight Availability and Irradiance Components from Direct and Global Irradiance*, Solar Energy, Vol.44, No.5, p271-289.


## APPENDIX - THEORY

The instantaneous luminance distribution throughout the sky vault may be described, relative to some reference point, using an expression due to Perez et al (1990):

$$\lambda_i = f(Z, \theta) \qquad \ldots(1)$$

where $Z$ is the zenith angle of the considered sky point, $\theta$ the angle between this point and the sun. Now, if we discretise the sky into a set of $p$ sky patches, of solid angle $\Phi$ and mean altitude $\bar{\gamma}$ then, given a diffuse horizontal irradiance $I_{dh}$, we may define a (relative luminance to absolute radiance) hemispherical normalisation factor:

$$\chi = I_{dh} \Big/ \sum_{i=1}^{p} \lambda_i \Phi_i \sin \bar{\gamma}_i \qquad \ldots(2)$$

From this we may calculate the absolute diffuse radiance $R$ of a sky patch:

$$R_i = \lambda_i \chi \qquad \ldots(3)$$

Alternatively the product of this absolute radiance and a diffuse luminous efficacy $\eta_d$ (say using the model due to Perez et al [4]), defines an absolute diffuse luminance $L$ distribution: $L_i = R_i \eta_d$ .

In *GenCumulativeSky* we use the discretisation scheme due to Tregenza in which the vault is split into seven azimuthal strips in which the azimuthal range of the composite patches tends to increase towards the zenith (12°, 12°, 15°, 15°, 20°, 30°, 60°), at which there is a single patch so that there are 145 in total. Each patch subtends a similar solid angle, as given by the azimuthal range $\Delta\alpha$ (radians) of the $i$th patch and the corresponding maximum and minimum heights of elevation $\gamma$:

$$\Phi_i = \Delta\alpha_i \left(\sin \gamma_{i,\max} - \sin \gamma_{i,\min}\right) \qquad \ldots(4)$$

The cumulative radiance ($Whm^{-2}Sr^{-1}$) / luminance ($lmh.m^{-2}Sr^{-1}$) for a given period is simply the aggregation of the instantaneous results within these patches.

Now, if for computational efficiency we wish to describe a global sky radiance distribution, we must increase the radiance of the patch in which the sun is located (that for which the angular distance between the sun and the patch centroid is the least): $R_{i,s} = I_b / \Phi_i$ . Again, for a luminance distribution the numerator should be multiplied by a beam luminous efficacy.

Alternatively, the properties of the full / binned set of suns should be defined. In this case we simply determine the angular coordinates (as a unit 3D vector) as well as its radiance, using an identical expression to that for the global sky, but substituting the solid angle with that of the solar disc (~0.222Sr, based on a half angle of $0.2665^{o}$).